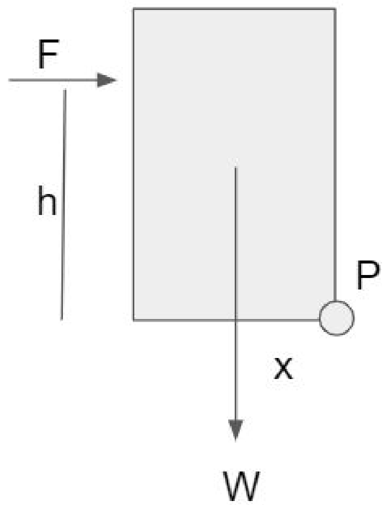
# WJEC (Wales) Physics A-level

## Topic 4.C: The Physics of Sports

Notes

## Centre of Gravity and Toppling

When an **object's centre of mass is outside of its base it topples**. This can be seen using moments about the point it is rotating.




The moment about P is the component of $Fh$ perpendicular to P minus the component of $Wx$ perpendicular to P, so the box will rotate clockwise about P if $F > Wx/h$. The **wider the base of an object , the more stable it is**.

In rugby, the player performing the tackle keeps a low centre of mass with their feet wide apart to increase their stability since the force of the impact will be close to their centre of mass and cause a small moment. The player being tackled, however, experiences the same force but further from their higher centre of mass - causing them to topple as they are much **less stable**.

We can use moments to calculate the moment about a joint in the human body when a person lifts a weight, by multiplying the distance from the joint and the weight together. By considering the forces needed to move a given weight we see that the force exerted by the muscles to do so is less when the distance between the pivot point and the is increased.

## Coefficient of Restitution

The coefficient of restitution, $e$, is the ratio of the **relative speed after a collision and the relative speed before a collision**.

$e$ usually takes values between 0 and 1 with the following meanings:

- $e$ = 1: The collision was perfectly **elastic** and no kinetic energy was dissipated.
- $0 < e < 1$: The collision was inelastic, some kinetic energy was dissipated
- $e$ = 0: The collision was perfectly **inelastic** - all of the kinetic energy was dissipated.

Suppose we drop an object from a height $H$ and it bounces to a height $h$ then the velocity at which it makes contact with the ground is given by:

$$v_{initial} = \sqrt{2gH}$$

where the SUVAT $v^2 = u^2 + 2as$ was used. Similarly, we can use the height reached after bouncing to calculate the velocity of the object the instant it has bounced and begins to travel upwards. This means that in the SUVAT 'v' is zero and we are solving for 'u', which gives:

$$v_{after} = \sqrt{2gh} .$$

Hence,

$$e = \frac{v_{after}}{v_{initial}} = \sqrt{\frac{h}{H}} .$$

## Moment of Inertia

Much like in the equation $F = ma$ mass is a measure of how difficult it is to accelerate an object, the moment of inertia, $I$, is a **measure of how difficult it is to change the rotational speed** of an object. It is calculated using:

$$\sum m_i {r^2}_i$$

where $m_i$ is the ith mass element and $r_i$ is the distance of that element to the **axis of rotation.**

Calculating the moment of inertia for common objects usually involves integration, and often non-cartesian coordinates so it is out of the scope of this course to do so but common moments of inertia are:

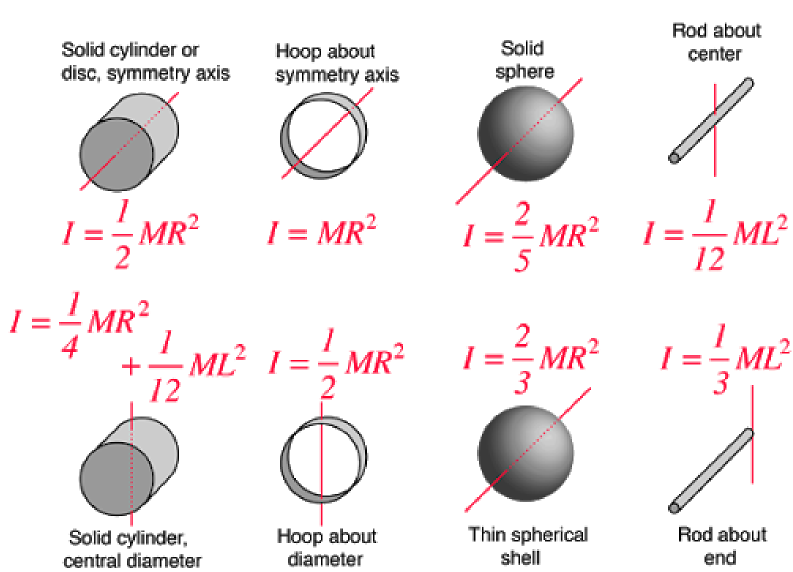



Image source: http://hyperphysics.phy-astr.gsu.edu/hbase/mi.html

# Rotational Dynamics

In linear dynamics we know that acceleration is the rate of change of velocity. In rotational movement we have that **angular acceleration**, , is the rate of **change of rotational velocity** and it is given by:

$$= \frac{_2 - _1}{t}$$

where $t$ is the time taken to go from the initial angular velocity, $_1$, to the final angular velocity, $_2$ .

Instead of force, used in linear dynamics to relate mass and acceleration, we use **torque** to measure the force required to cause angular acceleration, it is given by:

$$= I$$

or

$$= F r sin()$$

where $F$ is the linear force acting a distance $r$ from the axis of rotation at an angle .

We also have a rotational analogue to momentum from linear dynamics which is **angular momentum**:

$$L = I .$$

**Angular momentum is conserved** in a closed system that is not subject to external forces, and as a result we can use it to solve various problems in sport, for example when ice skaters are spinning they have a conserved angular momentum which means they can increase their angular velocity by decreasing their moment of inertia. They decrease their moment of inertia by bringing their arms closer to their centre (the axis of rotation) which reduces the amount their arms contribute to the total moment of inertia and as a result angular velocity must increase to keep their angular momentum the same.

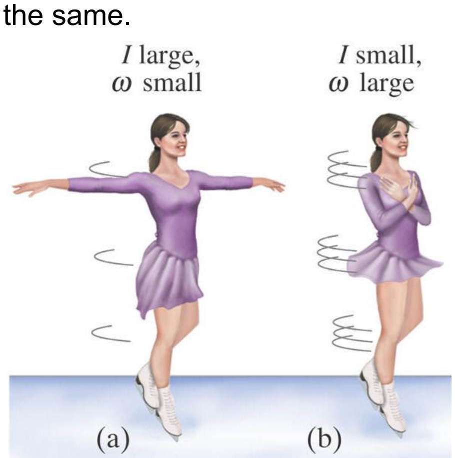



Image source: http://ffden-2.phys.uaf.edu/webproj/211_fall_2014/Ariel_Ellison/Ariel_Ellison/Angular.html

Much like we have these rotational analogues for non-rotational motion above, we also have **rotational kinetic energy** which is given by:

$$Rotational\ KE = \frac{1}{2} I ? \omega ?^2 .$$

We can combine this with the conservation of energy, linear kinetic energy and potential energy to solve problems relating to sport.

To conclude, here is a table showing linear quantities and their rotational counterparts:

| Linear | Rotational |
|---|---|
| Displacement, s | Angle, |
| Velocity, v | Angular velocity, |
| Acceleration, a | Angular acceleration, |
| Force, F | Torque, |
| Momentum, p | Angular momentum, $L$ |

## Projectile Motion

As the name would suggest, projectile motion is concerned with the motion of objects thrown/projected into the air with the **only force acting upon them**, other than the initial force, **being gravity**. We can use the SUVAT equations to solve problems involving projectile motion. For example, the maximum height, $H$, attained by an object projected at an angle and velocity $v$:

$$v^2 = u^2 + 2as$$

with $s = H$ , $a = -g$, $v_y = 0$, $u_y = vsin()$ since we only care about **vertical motion** and the vertical velocity will be zero at the maximum height. Hence,

$$H = \frac{(vsin())^2}{2g} .$$

Similarly, the time the projectile is in the air:

$$v = u + at$$

with $v_y = 0$, $u_y = vsin()$ and $a = -g$ which represents the projectile rising to its maximum height, where $v_y = 0$ since gravity has slowed it down:

$$t_{max} = \frac{vsin()}{g}$$

and since the motion is symmetric, the time taken to go from the maximum to the ground is simply $2t_{max}$:

$$t_{total} = 2\frac{vsin()}{g}.$$

Now we know $t_{total}$ we can use it to find the distance, $R$, the object travels in the horizontal direction:

$$s = ut$$

With $u_x = vcos()$ and $t = t_{total} = 2\frac{vsin()}{g}$ giving:

$$R = 2\frac{v^2 sin()cos()}{g}$$

$$R = \frac{v^2 sin(2)}{g}.$$

Using these equations we can easily calculate the motion of a javelin, given a few initial conditions.

## Bernoulli's Equation

Bernoulli's equation **relates the pressure, the static pressure, the density and the velocity of a fluid**, it is as follows:

$$p = p_0 - \frac{1}{2}v^2$$

where $p$ is the pressure, $p_0$ is the static pressure,  is the density of the fluid and $v$ is the velocity.

**Bernoulli's equation** has aspects in sport. For example how balls move through the air and how F1 cars can increase their downforce (and grip) by changing the velocity of the air at the top and bottom and thus pushing the car downwards.

## Drag

When an object moves through a fluid (liquid or gas) it **experiences a resistance to motion known as drag**. One can calculate the magnitude of the drag force using

$$F_D = \frac{1}{2}v^2 AC_D$$

where $F_D$ is the drag force,  is the density of the fluid, $v$ is the velocity of the object moving relative to the fluid, $A$ is the cross sectional area of the object and $C_D$ is the **drag coefficient** which is a dimensionless constant that describes that resistance to motion in a fluid.